Russian Style (Lack of) Randomness

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Russian Style (Lack of) Randomness

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Based on joint works with **A. Biryukov** (uni.lu), **X. Bonnetain** (SECRET), **A. Udovenko** (uni.lu), and **S. Tian** (SECRET+China)

September 11, 2019

No secret structure was enforced during construction of the S-box. At the same time, it is obvious that for any transformation a lot of representations are possible (see, for example, a lot of AES S-box representations).

VS.

$$\mathscr{T}_{\kappa,s} : \begin{cases} \mathbb{F}_{2^8} & \to \mathbb{F}_{2^8} \\ 0 & \mapsto \kappa(0)\,, \\ \left(\alpha^{2^m+1}\right)^j & \mapsto \kappa(2^m - j), \ \text{for } 1 \le j \le 2^m - 1\,, \\ \alpha^{i+(2^m+1)j} & \mapsto \kappa(2^m - i) \oplus \left(\alpha^{2^m+1}\right)^{s(j)}, \ \text{for } 0 < i, 0 \le j < 2^m - 1\,. \end{cases}$$

Outline

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Kuznyechik/Streebog

Streebog

Type: Hash function

Publication: 2012

Kuznyechik

Type: Block cipher

Publication: 2015

Kuznyechik/Streebog

Streebog

Type	Hash function
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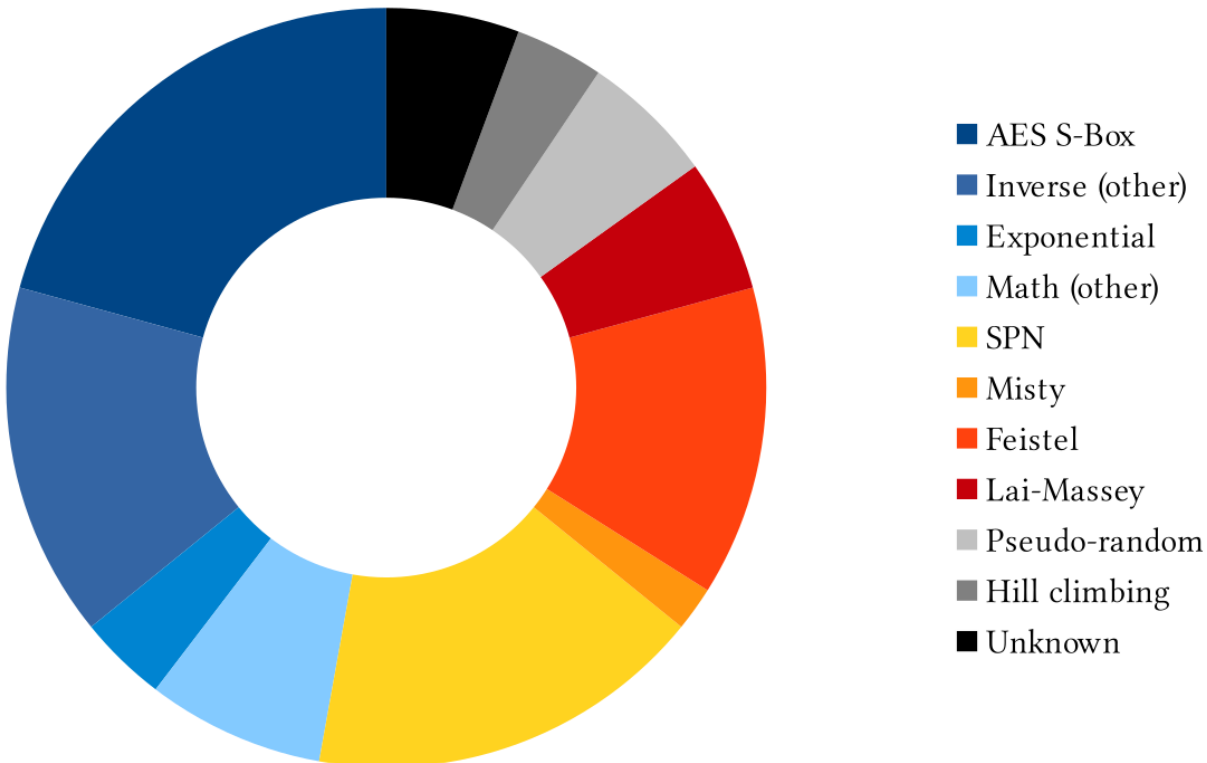
Common ground

- Both are standard symmetric primitives in Russia.
- Both were designed by the FSB (TC26).
- Both use the same 8×8 S-Box, π.

S-Boxes

Definition (S(ubstitution)-box)

An S-box $S : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$ is a small **non-linear** function operating on a small block size (typically $n \in \{4, 8\}$) which **can** be specified via its lookup table.



Cryptographic strength **vs.** Implementation efficiency

The Russian S-box

π' = (252, 238, 221, 17, 207, 110, 49, 22, 251, 196, 250, 218, 35, 197, 4, 77, 233, 119, 240, 219, 147, 46, 153, 186, 23, 54, 241. 187, 20, 205, 95, 193, 249, 24, 101, 90, 226, 92, 239, 33, 129, 28, 60, 66, 139, 1, 142, 79, 5, 132, 2, 174, 227, 106, 143, 160, 6, 11, 237, 152, 127, 212, 211, 31, 235, 52, 44, 81, 234, 200, 72, 171, 242, 42, 104, 162, 253, 58, 206, 204, 181, 112, 14, 86, 8, 12, 118, 18, 191, 114, 19, 71, 156, 183, 93, 135, 21, 161, 150, 41, 16, 123, 154, 199, 243, 145, 120, 111, 157, 158, 178, 177, 50, 117, 25, 61, 255, 53, 138, 126, 109, 84, 198, 128, 195, 189, 13, 87, 223, 245, 36, 169, 62, 168, 67, 201, 215, 121, 214, 246, 124, 34, 185, 3, 224, 15, 236, 222, 122, 148, 176, 188, 220, 232, 40, 80, 78, 51, 10, 74, 167, 151, 96, 115, 30, 0, 98, 68, 26, 184, 56, 130, 100, 159, 38, 65, 173, 69, 70, 146, 39, 94, 85, 47, 140, 163, 165, 125, 105, 213, 149, 59, 7, 88, 179, 64, 134, 172, 29, 247, 48, 55, 107, 228, 136, 217, 231, 137, 225, 27, 131, 73, 76, 63, 248, 254, 141, 83, 170, 144, 202, 216, 133, 97, 32, 113, 103, 164, 45, 43, 9, 91, 203, 155, 37, 208, 190, 229, 108, 82, 89, 166, 116, 210, 230, 244, 180, 192, 209, 102, 175, 194, 57, 75, 99, 182).

*Screen capture of the specification of **Kuznyechik** (2015).*

Why we need S-box reverse-engineering

The security of a cipher hinges on its S-box: we need to know how it works!

Outline

A Key Tool for Analysing S-Boxes

Linear Approximations Table (LAT)

The LAT of $S : \{0,1\}^n \to \{0,1\}^n$ is a $2^n \times 2^n$ matrix such that

$$\begin{aligned} \mathrm{LAT}_S[a,b] &= \sum_{x \in \mathbb{F}_2^n} (-1)^{a \cdot x \oplus b \cdot S(x)} \\ &= 2 \times \left(\#\{x \in \mathbb{F}_2^n, a \cdot x = b \cdot S(x)\} \right) - 2^n \, . \end{aligned}$$

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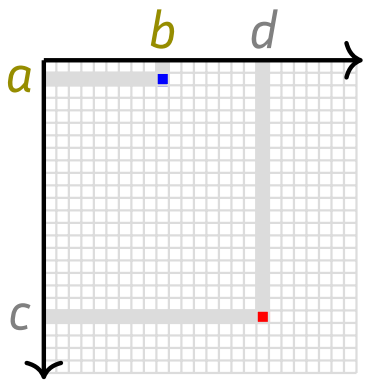
"Jackson Pollock" Representation[1]



$$\left|\mathrm{LAT}_S[a,b]\right| = 0$$

$$\left|\mathrm{LAT}_S[c,d]\right| \geq 20$$

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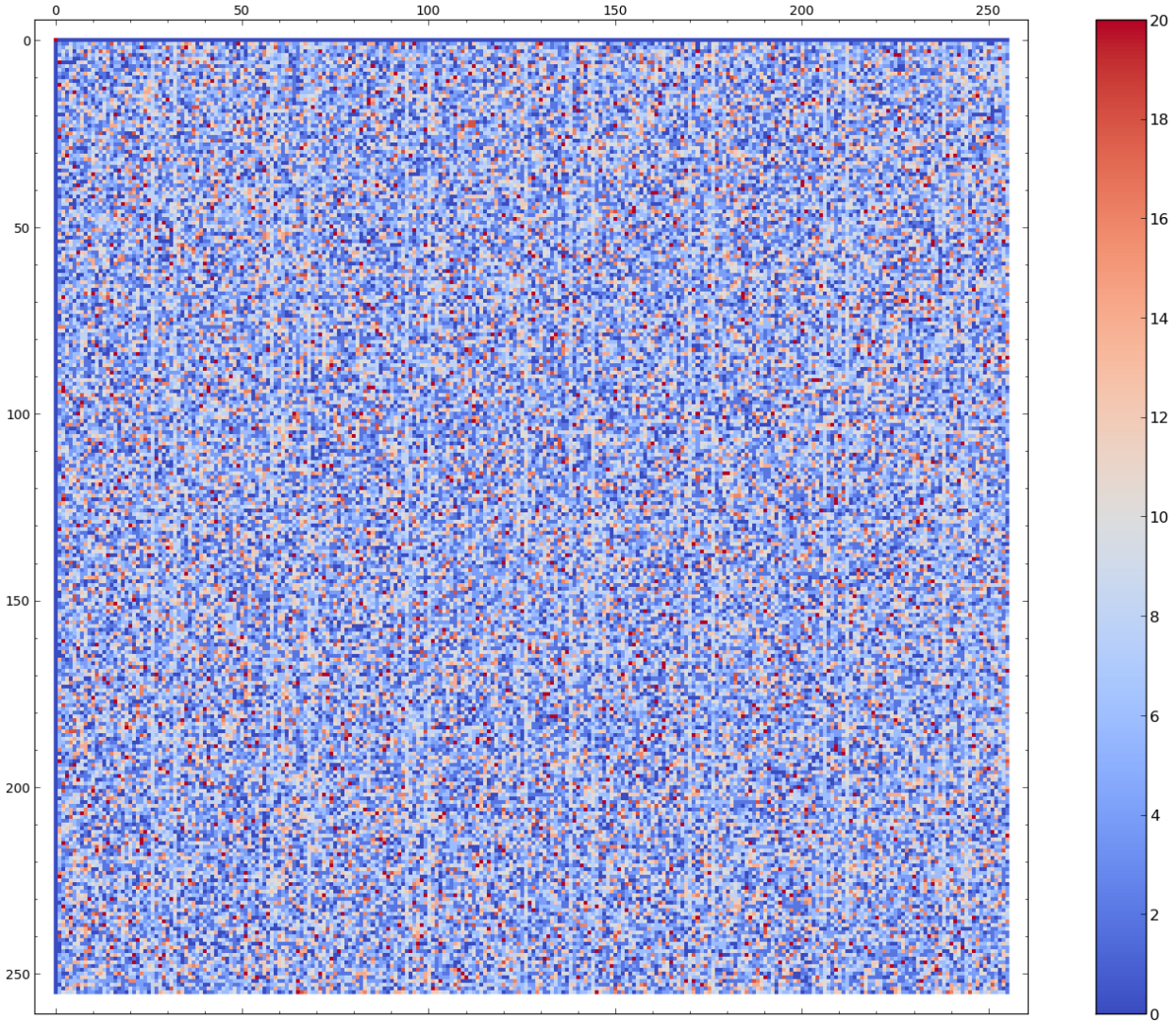
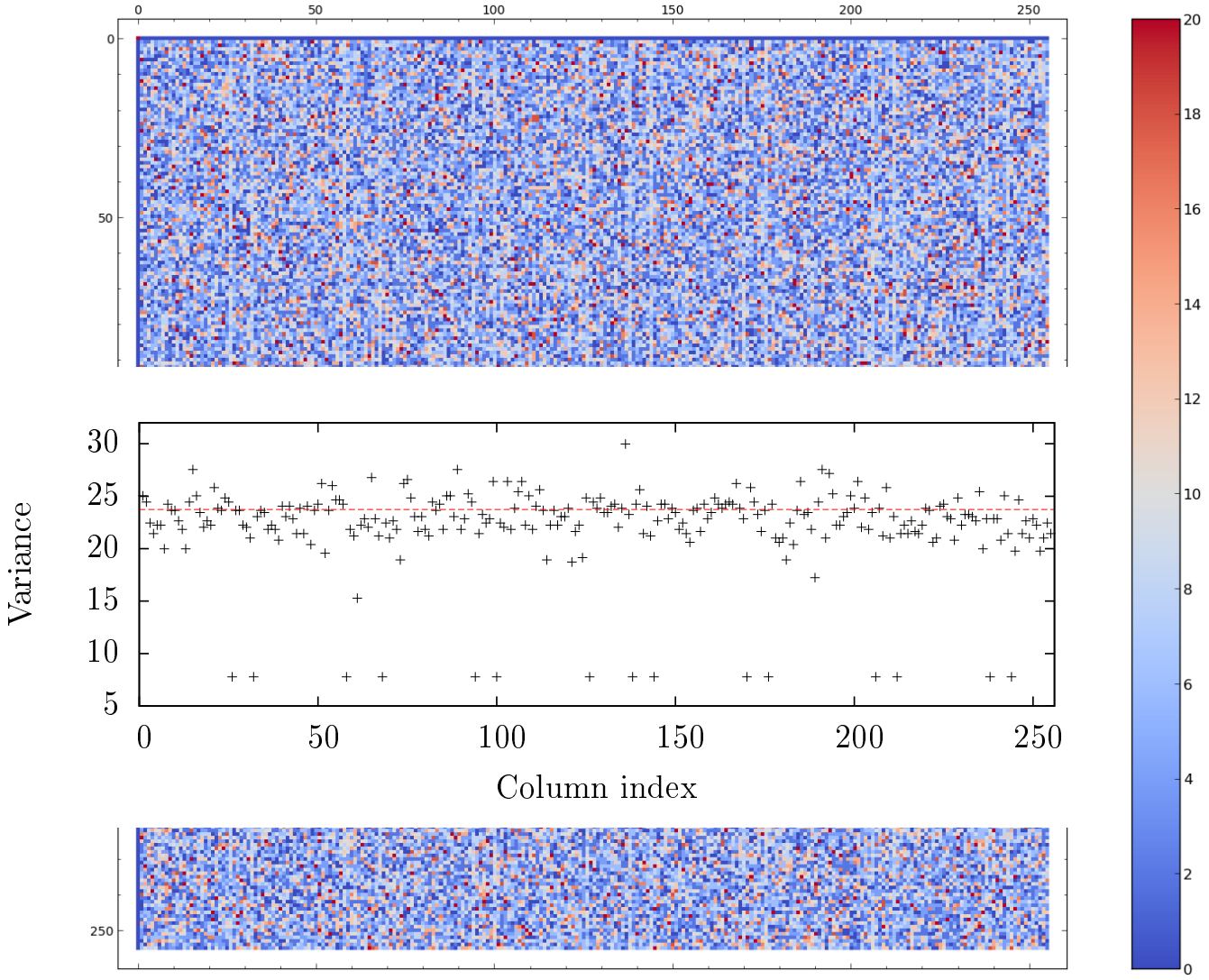
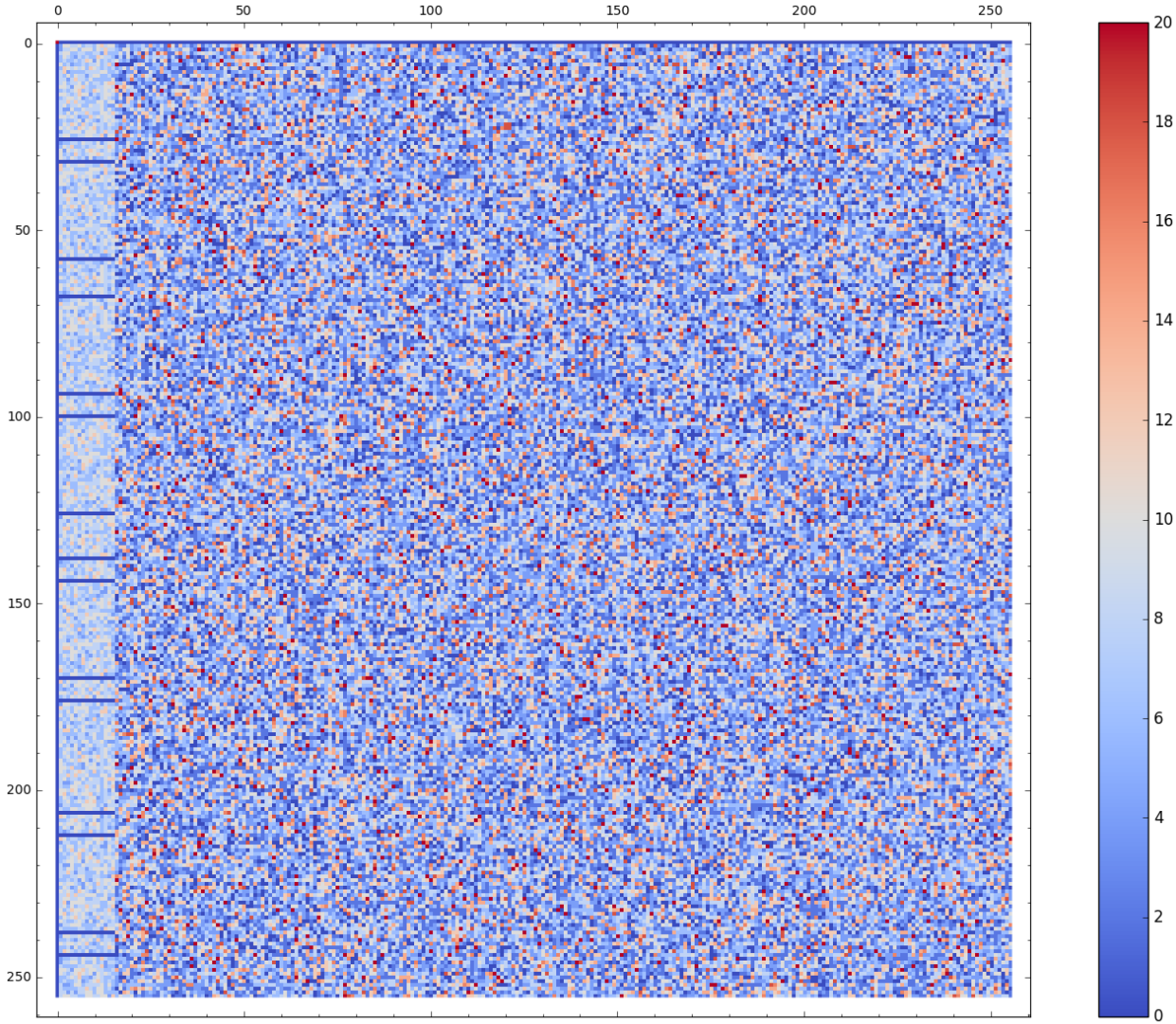
The LAT of π



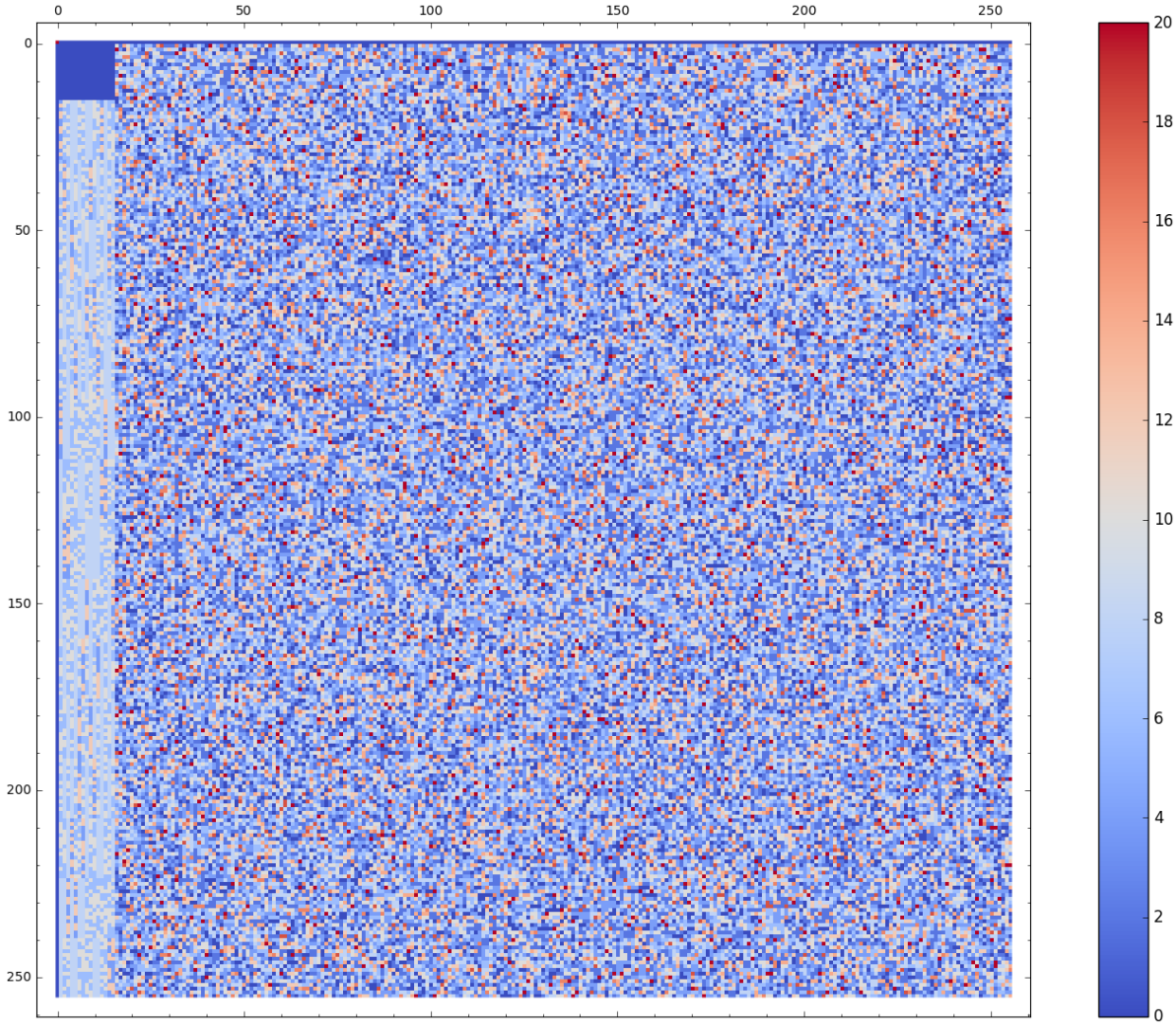
The LAT of π


0
50
100
150
200
250
Variance
30
25
20
15
10
5
Column index
20
18
16
14
12
10
8
6
4
2
0

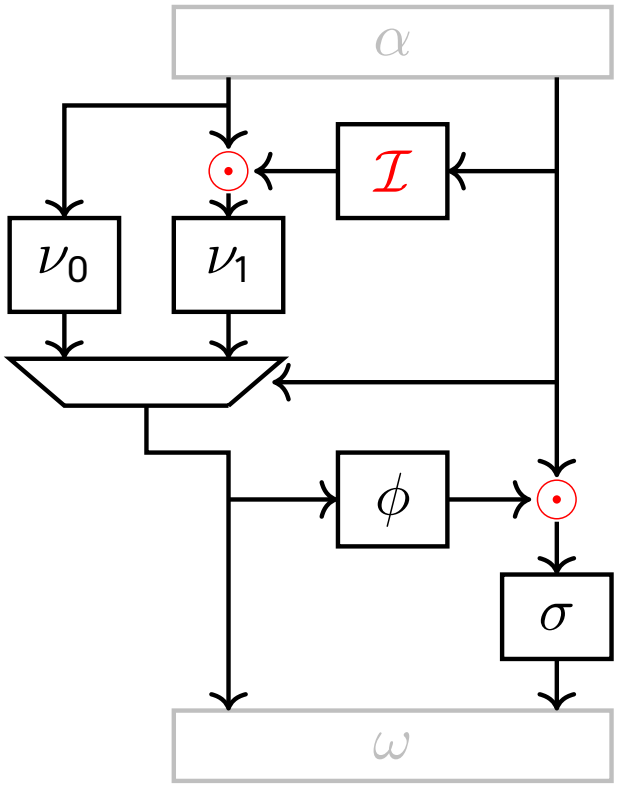
The LAT of π (reordered columns)



The LAT of $\ell_2 \circ \pi \circ \ell_1$



First Complete Decomposition of π



$\odot$ Multiplication in $\mathbb{F}_{2^4}$

$\mathcal{I}$ Inversion in $\mathbb{F}_{2^4}$

ν_0, ν_1, σ 4×4 permutations

ϕ 4×4 function

α, ω Linear permutations

Conclusion for Kuznyechik/Streebog?

The Russian S-Box was built like a strange Feistel...

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Belarussian inspiration

- The last standard of Belarus (BelT) uses an 8-bit S-box,
- somewhat similar to π...
- ... based on a finite field exponential!
- We deduced another decomposition[2] but not a satisfactory one.

[2] L. Perrin, A. Udovenko. *Exponential S-boxes: a link between the S-boxes of BelT and Kuznyechik/Streebog.* IACR ToSC. 2016.

Timeline

July 2012	GOST standardization of Streebog
Aug. 2013	RFC for Streebog (RFC6986)
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Oct. 2018	ISO standardization of Streebog (ISO 10118-3)

A Third and Final Decomposition: the TKlog

π is a TKlog!

A TKlog operates on $\mathbb{F}_{2^{2m}}$ and uses:

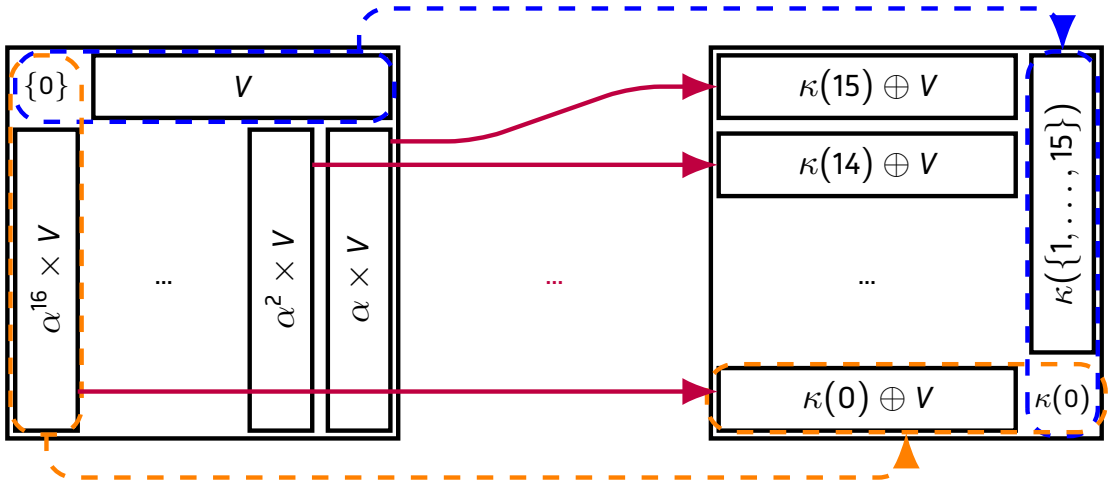
- α: a generator of $\mathbb{F}_{2^{2m}}$,
- κ: an affine function $\mathbb{F}_2^m \to \mathbb{F}_{2^{2m}}$ with $\kappa(\mathbb{F}_2^m) \oplus \mathbb{F}_{2^m} = \mathbb{F}_{2^{2m}}$,
- s: a permutation of $\mathbb{Z}/(2^m - 1)\mathbb{Z}$.

The corresponding TKlog is denoted $\mathscr{T}_{\kappa,s}$ and it works as follows:

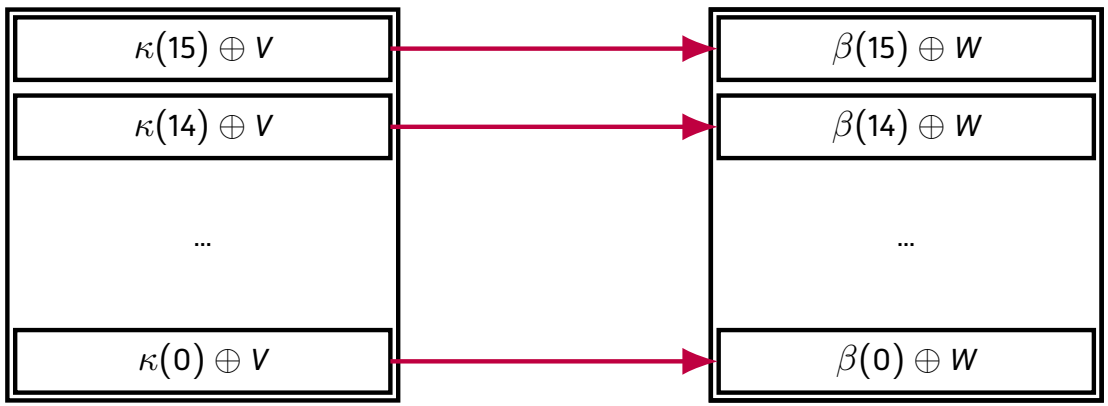
$$\begin{cases} \mathscr{T}_{\kappa,s}(0) & = \kappa(0)\,, \\ \mathscr{T}_{\kappa,s}\left((\alpha^{2^m+1})^j\right) & = \kappa(2^m - j), \text{ for } 1 \le j \le 2^m - 1\,, \\ \mathscr{T}_{\kappa,s}\left(\alpha^{i+(2^m+1)j}\right) & = \kappa(2^m - i) \oplus \left(\alpha^{2^m+1}\right)^{s(j)}, \text{ for } 0 < i, 0 \le j < 2^m - 1\,. \end{cases}$$

Cosets to Cosets

Russia's π



Backdoored S-box



Outline

From the Designers, at ISO

questioned is the S-box π. This S-box was chosen from Streebog hash-function and it was synthesized in 2007. Note that through many years of cryptanalysis no weakness of this S-box was found. The S-box π was obtained by pseudo-random search and the following properties were taken into account.

[...]

No secret structure was enforced during construction of the S-box. At the same time, it is obvious that for any transformation a lot of representations are possible (see, for example, a lot of AES S-box representations).

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Everything is wrong (except for the AES bit).

A Key Concept: “Anomalies” (1/2)

How “far” is the behaviour of a specific S-box π from that of a “random S-box”?

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First idea

What is the probability of a specific property of the S-box?

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First idea

What is the probability of a specific property of the S-box?

Does not work!

The properties we consider are not intended for that purpose.

- $\Pr\left[\text{Differential uniformity} = 8\right] \approx 2^{-16.15} \approx (72000)^{-1}$
- $\Pr\left[\text{Differential uniformity} = 6\right] \approx 2^{-164.5} \approx (3 \cdot 10^{49})^{-1}$

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1. Choose a **fined grained** property (LAT max/# occurrences of the max, implementation complexity...) with a value in a partially ordered set (i.e. a number)

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Negative Anomaly

$$\overline{\mathcal{A}}(\pi) = -\log_2\left(\frac{\#\text{worse S-boxes}}{(2^n)!}\right)$$

Negative Anomaly

$$\mathcal{A}(\pi) = -\log_2\left(\frac{\#\text{better S-boxes}}{(2^n)!}\right)$$

Some Very High Anomalies

Informally (more details at AC'19[3]), a high positive anomaly for a **property** means:

- a *random* S-box is unlikely to have a **property** "at least as pronounced" as π;
- we need to generate about $2^{\mathcal{A}}$ random S-boxes to get one where the **property** is this strong.

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Anomalies of π.

Statistical			**Structural**	
Differential	Linear	Boomerang	TU_4	TKlog
80.6†	34.4	14.2	201.1	1601.5

† This anomaly might be overestimated.

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Kolmogorov Anomaly (2/2)

The "**Shannon effect**":[4]

> *"almost all functions" of n arguments have "an almost identical" complexity which is asymptotically equal to the complexity of the most complex function of n arguments.*

Indeed, specific structures (TKlog, TU-decompositions...) have a very high anomaly...

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... but not *really* an anomaly. Can we do better?

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Kolmogorov Anomaly (2/2)

```
p(x){unsigned char*k="@`rFTDVbpPB
vdtfR@\xacp?\xe2>4\xa6\xe9{z\xe3q
5\xa7\xe8",a=2,l=0,b=17;while(x&&
(l++,a^x))a=2*a^a/128*29;return l
%b?k[l%b]^k[b+l/b]^b:k[l/b]^188;}
```

165 ASCII characters that fit on **7** bits: this program is **1155**-bit long.

`https://codegolf.stackexchange.com/questions/186498/proving-that-a-russian-cryptographic-standard-is-too-structured`

Kolmogorov Anomaly of π

The probability that a random 8-bit S-box is as structured as π is at most equal to

$$2^{1155-1684} = 2^{-529}.$$

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